

# Analytic Model of Solar Power Plant with a Stirling Engine

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**Abstract**—An analytic model is proposed of a solar power plant (SPP) with a Stirling engine that is based on the isothermal model of the Stirling engine (SE) working process and is improved by account for the actual heat exchange, the hydraulic and mechanical losses in the SE, the losses in the electric generator, and also the basic parameters of the solar radiation concentrator.

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Theoretically, the ideal Stirling cycle has the maximal possible efficiency for a given temperature differential, equal to the efficiency of the Carnot cycle. However the real working process in the Stirling engine differs from the ideal process, and its effectiveness is considerably lower. This is due to the irreversibility of the actual thermodynamic processes, the deviation of the working processes in the working chambers of the Stirling engine from isothermality, the nonideality of the heat exchange processes in the heater, the regenerator, and the cooler, and also the heat losses to the ambient medium.

The program for calculating the characteristics of the Stirling engine is based on the Schmidt isothermal model that is improved by accounting for the heat transfer, the hydraulic resistances, the nonideality of the regenerator, the mechanical losses, and the losses in the electric generator [1, 2]. Moreover, the sinusoidal motion of the pistons is replaced by the equations of the actual motion of the pistons with account for the specific drive mechanism. In the calculation of the solar power plant as a whole, the losses in the concentrator–receiver system are taken into account.

Figure 1 presents the scheme of the 5-kW Stirling engine with a swashplate drive that was developed at FTI NPO “Fizika-Solntse” of the Academy of Sciences, Republic of Uzbekistan. The algorithm for the calculation of the engines of this type is presented later.

The piston travel is defined by the expression

$$h = 2Rs \sin(\alpha), \quad (1)$$

where  $\alpha$  is the swashplate inclination angle; and  $R_s$  is the radius of the swashplate.

The instantaneous piston position 1 is defined by the expression

$$Z_1 = 0.5h(1 + \cos(\omega t)). \quad (2)$$

The reference point is  $\omega t = 0$ , the piston is at the BDC point, and  $VG = \max$ .

The volume of the hot chamber of the first cylinder is

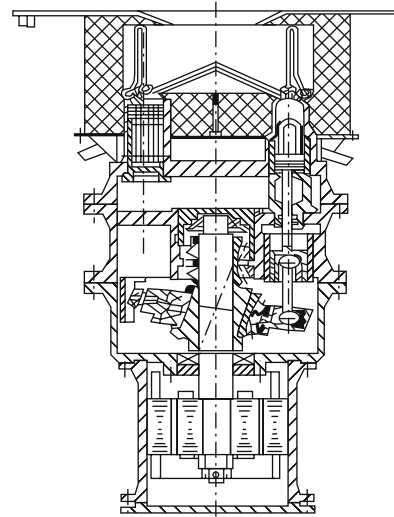
$$V_g = 0.785 Dcl^2 (Z_1 + Hgm); \quad (3)$$

the volume of the cold chamber of the adjacent (second) cylinder is

$$V_x = 0.785 (Dcl^2 - Dst^2) (h + Hxm - Z_2), \quad (4)$$

where  $Hgm$  and  $Hxm$  are the piston clearances at the TDC and BDC points;  $Dcl$  is the diameter of the cylinder;  $Dst$  is the diameter of the rod; and  $Z_2 = 0.5h(1 + \cos(\omega t + \pi/2))$ .

The calculation of the working fluid pressure function for the given initial temperatures of the working



**Fig. 1.** Solar Stirling engine with swashplate drive.

chambers with the absence of hydraulic resistances in the heat exchangers is performed as follows:

$$P(\omega t) = P_{sr}(1 - A^2)/(1 + A \cos(\omega t - B)), \quad (5)$$

where  $A_1 = (\tau^2 + K^2 + 2\tau K \cos(\pi/2))^{0.5}$ ;  $A_2 = \tau + K + 2(Xh\tau + Xc + 2(Xr\tau)/(1 + \tau))$ ;  $A = A_1/A_2$ ;  $B_1 = K \sin(\pi/2)/A_1$ ;  $B = A \sin(B_1)$ ;  $\tau = T_{xs}/T_{gs}$ ;  $T_{gs}$ ,  $T_{xs}$  are the temperatures of the walls of the heater and the cooler.

The calculation of the working fluid masses in the hot and cold chambers and in the heat exchangers of the engine is performed on the basis of the condition that the working fluid is an ideal gas:

$Mg(\omega t) = P(\omega t)Vg(\omega t)/(RTg)$  is the working fluid mass in the hot chamber;

$Mh(\omega t)P(\omega t)V_1Xh/(RTg)$  is the working fluid mass in the heater;

$Mr(\omega t) = 2P(\omega t)V_1Xr/(R(Tg + Tx))$  is the working fluid mass in the regenerator;

$Mc(\omega t)P(\omega t)V_1Xc/(RTx)$  is the working fluid mass in the cooler;

$Mx(\omega t) = P(\omega t)Vx(I)/(RTx)$  is the working fluid mass in the cold chamber.

The calculation of the heat transfer coefficients in the heat exchangers (heater and cooler) is based on the assumption of quasistationarity of the heat exchange processes, for the instantaneous values of the gas velocity in the heat exchanger channels,

$$\text{Reh}(\omega t) = \Delta\omega t N L h D h \text{ABS}(dMg(\omega t) + 0.5dMh(\omega t))/(V_1\mu_h); \quad (6)$$

$$\text{Rec}(\omega t) = \Delta\omega t N L c D c \text{ABS}(dMx(\omega t) + 0.5dMc(I))/(V_1\mu_c). \quad (7)$$

For the laminar flow regime with  $\text{Re} < 2200$ , the Nusselt number for each heat exchanger is calculated using the expression [3]

$$\text{Nu}(\omega t) = 0.142 \text{Re}(\omega t)^{0.33}. \quad (8)$$

For the transition interval, the formulas for calculating the Nusselt number are derived from the tabulated values:

for  $\text{Re}(\omega t) > 2200$  and  $< 5600$

$$\text{Nu}(\omega t) = 32.8 \times 0.43429 \text{ALOG} \times (\text{Re}(\omega t)) - 107.57; \quad (9)$$

for  $\text{Re}(\omega t) > 5600$  and  $< 10000$

$$\text{Nu}(\omega t) = 48.07 \times 0.43429 \text{ALOG} \times (\text{Re}(\omega t)) - 164.88. \quad (10)$$

If  $\text{Re}(\omega t) > 10000$ , i.e., for the turbulent regime

$$\text{Nu}(\omega t) = 0.0175(\text{Re}(\omega t))^{0.8}. \quad (11)$$

The calculation of the heat transfer in the cooler is performed similarly. As a result, we obtain the magni-

tudes of the instantaneous and cycle-average heat transfer coefficients in the heater and the cooler:

$$\alpha_h(\omega t) = \text{Nuh}(\omega t)\lambda_h/Dh; \quad (12)$$

$$\alpha_c(\omega t) = \text{Nuc}(\omega t)\lambda_c/Dc;$$

$$\alpha_{hsr} = \sum \alpha_h(\omega t)/(2\pi/\Delta\omega t); \quad (13)$$

$$\alpha_{csr} = \sum \alpha_c(\omega t)/(2\pi/\Delta\omega t).$$

For the calculation of the average temperatures in the working chambers of the engine, we arbitrarily specify the initial magnitude of the heat that is supplied to the cycle:

$$Q_1 = 0.45 P_{sr} N V_1. \quad (14)$$

Then the magnitudes of the temperature differentials on the heat exchange surfaces are defined by the expressions

$$\Delta Th = Q_1 Dh/(\alpha_h 4 Xh V_1); \quad (15)$$

$$\Delta Tc = Q_1 \tau Dc/(\alpha_c 4 Xc V_1). \quad (16)$$

The gas temperatures in the hot and cold chambers of the engine are determined by the relations  $Tg = Tgs - \Delta Th$ ,  $Tx = Txs - \Delta Tc$ , and their ratio  $\tau_r = Tx/Tg$ .

The working fluid pressure functions in the cold and hot chambers of the engine with account for the real heat exchange and the actual hydraulic resistances are determined by the expressions

$$Px(\omega t) = P(\omega t)(1 - \Delta P(\omega t) \times ((1 + \cos(\omega t))\tau r + 2Xh\tau r + 2Xr\tau r/(1 + \tau r))/(A_2 P_{sr})); \quad (17)$$

$$Pg(\omega t) = Px(\omega t) + \Delta P(\omega t), \quad (18)$$

where  $\Delta P(\omega t)$  is the overall hydraulic resistance in the channels of the heat exchangers and the connecting lines, which can be determined from the known relations for the instantaneous working fluid velocities.

The calculation of the indicated work in the hot and cold chambers of the engine:

$$Lig = \int Pg(\omega t) dVg(\omega t); \quad (19)$$

$$Lix = \int Px(\omega t) dVx(\omega t). \quad (20)$$

The regenerator effectiveness  $\eta_r$  is determined using the technique described in [4].

The heat that is supplied to the hot chamber of the engine is determined by the expression

$$Q = Lig + Qr(1 - \eta_r), \quad (21)$$

where  $Qr = CpGr(Tg - Tx)$  is the heat that is supplied to and removed from the regenerator per cycle, and  $Gr$  is the gas flowrate in the regenerator.

Then the calculation is performed by the successive approximation method. For example, if  $\text{ABS}(Q_1 - Q)Q_1 >$

$> 0.001$ , i.e., the initial and obtained values differ by a magnitude that is larger than 0.001, the calculation is repeated until the accuracy of the calculation satisfies the specified condition. In this case the value  $(Q_1 + Q)/2$  is specified as the initial value  $Q_1$ . As a result, we obtain:

$$\text{the indicated work per cycle } Li = Lig + Lix; \quad (22)$$

$$\text{the indicated power } Ni = Lin; \quad (23)$$

$$\text{the indicated engine efficiency } \eta_i = Li/Q. \quad (24)$$

For the presentation of the results of the calculations, it is helpful to use the specific power, which is defined by the expression

$$PM = Ni/(PSR(1 + A)V_1n). \quad (25)$$

We shall use the dimensionless power for the calculation of the indicated power and the determination of the parameters of the heat exchangers for the engines of analogous design, in the present case of the alpha type, but with differing working volume.

The friction process in the friction assemblies of the cylinder–piston group of the Stirling engine takes place with reciprocating motion of the piston rings, which are made of antifriction materials, relative to the steel surface of the cylinder in the conditions of the absence of a lubricant. A combination of guide and sealing rings is usually installed on the pistons. The friction forces in the sealing rings of the SE arise because they are pushed against the cylinder wall by the forces of the gas pressure and by the elastic force of the deformation of the expander in the rectilinear reciprocating motion of the piston. The friction forces in the guide rings arise because they are pushed against the cylinder wall by the lateral components of the forces from the drive mechanism.

The mechanical losses on friction in the piston rings are calculated using the technique described in [5].

The mechanical power on the engine shaft with account for the losses in the piston–cylinder group and the drive is defined by the expression

$$Ne = 4(Ni - Ntr)\eta_{dr}. \quad (26)$$

The mechanical (effective) efficiency of the engine is determined as

$$\eta_e = Ne/(4Q). \quad (27)$$

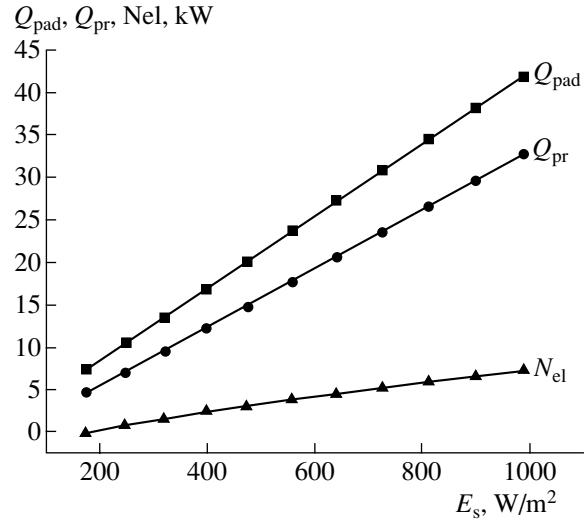
The electric power and efficiency of the engine are:

$$Nel = Ne/\eta_{gen}; \quad (28)$$

$$\eta_{el} = Nel/(4Q), \quad (29)$$

where  $\eta_{gen} = f(Nel)$ . The study of the characteristics of the electric generator was performed earlier on the one-kW and five-kW power levels [6, 7].

The efficiency of the solar power plant as a whole is defined by the ratio of the generated electric power to the solar flux incident on the concentrator:



**Fig. 2.** Dependences of the power of the solar radiation flux that is incident on the concentrator, the thermal power that is transferred to the engine working fluid, and the generated electric power on the density of the direct solar radiation.

$$\eta_{SPP} = Nel/(EsFk). \quad (30)$$

For the calculation of the required concentrator area, it is necessary to consider the losses due to the accuracy of the concentrator tracking system, the alignment losses, the heat receiver losses, and also the density of the direct solar radiation.

The calculation of the torque is performed for each cylinder, and then, summing these values for each moment of time, we obtain the torque on the engine shaft. The force that acts on the shaft from a single piston is

$$Fkri(\omega t) = \Delta Pi(\omega t)tn(\alpha)(-\sin(\omega t)), \quad (31)$$

and the torque from a single piston is

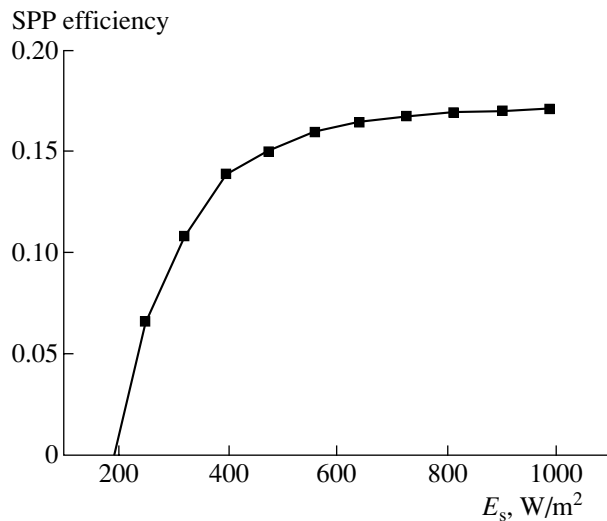
$$Mkri(\omega t) = \frac{Fkri(\omega t)Rs \cos(\alpha)}{\sqrt{\sin(\omega t + 0.5\pi)^2 + \cos(\alpha)^2 \cos(\omega t + 0.5\pi)^2}}. \quad (32)$$

The torque on the engine shaft is determined by the sum of the instantaneous values from all the pistons:

$$Mkr(\omega t) = \sum Mkri(\omega t). \quad (33)$$

The calculations were made for a solar power plant with a 5-kW Stirling engine. The following characteristics of the concentrator were adopted for the calculation: the concentrator area is 40 m²; the reflection coefficient is 0.85; the alignment losses and the tracking inaccuracy losses are 5%. The results of the calculation are presented in Figs. 2 and 3.

The results of the calculation agree with the experimental data that were obtained in the tests of a solar power plant with a V-160 engine rated at 7.5 kW. The



**Fig. 3.** Dependence of efficiency of SPP with 5-kW Stirling engine on the density of the direct solar radiation.

tests were performed at the solar test station at Almeria (Spain) [8]. The developed analytic technique can be used for the analysis and preparation of the basic data in the design of the solar power plant with a Stirling engine.

#### REFERENCES

1. Umarov, G.Ya., Trukhov, V.S., and Tursunbaev, I.A., *Raschet parametrov vnutrennego teploobmennogo kontura dvigatelya Stirlinga*, Tashkent: Fan, 1979.
2. Tursunbaev, I.A., Energy Balance of Autonomous Solar Power Plants with a Stirling Engine, *Proc. of Third International Conference: Fundamental and Applied Questions of Physics*, Tashkent, 2006, pp. 123–125.
3. Mikheev, M.A., *Osnovy Teploperedachi*, Moscow, 1956.
4. Kays, W.M. and London, A.L., *Compact Heat Exchangers*, Moscow, 1967.
5. Kenzhaev, I.G. and Tursunbaev, I.A., Analysis of the Friction Forces in the Components of the Cylinder–Piston Group and their Influence on the Performance of the Stirling Engine, *Proc of Third International Conference: Fundamental and Applied Questions of Physics*, Tashkent, 2006, pp. 82–84.
6. Trukhov, V.S., Tursunbaev, I.A., Orda, E.P., et al., Development of an Experimental Stirling Engine with a Gas Burner for an Autonomous Power Plant, *Proc. of Conference Dedicated to 60 Years of the Academy of Sciences RUz and FTI* (November 27–28, 2003), Tashkent, pp. 88–93.
7. Ye Hong, Wang Jun, Tursunbaev, I.A., et al., Modification and Performance Test of a 1 kW Alpha-Type Stirling Engine (Department of Thermal Science and Power Engineering, University of Science and Technology of China); source: Taiyangneng Xuebao, *Acta Ener-giae Solaris Sinica*, 2005, vol. 26, no. 5, pp. 703–707.
8. Blezinger, H., Continuous Operation of a Dish/Stirling Field on Plataforma Solar de Almeria. Plataforma Solar de Almeria (PSA), *Deutsche Forschungsanstalt fur Luft-und Raumfahrt*, Tabernas, Spain, 1994.